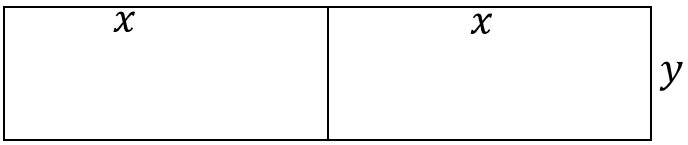
OPTIMISATION (MERIT)

Optimisation problems at Merit level usually require you to form an equation, eliminate variables, and involve harder differentiation. Sometimes drawing a diagram of the problem is useful.

Example: Two adjoining rectangular yards share a boundary. There is 60 m of fencing available for the boundaries. Calculate the maximum total area for the two yards.



- Let the dimensions of each yard be x metres by y metres.
- The fencing all around and down the middle is 60 m, hence $4x + 3y = 60$.
- The area of the two yards is: $A = 2x \cdot y$
- Before we differentiate A, we need to eliminate one of the variables.
- Since $4x + 3y = 60$, rearranging this gives $y = \frac{60-4x}{3}$.
- Substitute $y = \frac{60-4x}{3}$ into $A = 2x \cdot y$: $A = 2x \cdot \left(\frac{60-4x}{3}\right)$
- Now differentiate A using the product rule: $A' = 2 \cdot \left(\frac{60-4x}{3}\right) + (-4) \cdot 2x$
- Let $A' = 0$: $2 \cdot \left(\frac{60-4x}{3}\right) + (-4) \cdot 2x = 0$
- Solve: $\frac{120-8x}{3} - 8x = 0 \Rightarrow \frac{120-8x}{3} = 8x \Rightarrow 120 - 8x = 24x \Rightarrow x = 3.75$
- Substitute $x = 3.75$ into $A = 2x \cdot \left(\frac{60-4x}{3}\right)$: $A = 112.5$

Therefore the maximum total area for the two yards is 112.5 m^2.

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